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## Instantons at Angles

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### ABSTRACT

We interpret a class of  $4k$ -dimensional instanton solutions found by Ward, Corrigan, Goddard and Kent as four-dimensional instantons at angles. The superposition of each pair of four-dimensional instantons is associated with four angles which depend on some of the ADHM parameters. All these solutions are associated with the group  $Sp(k)$  and are examples of Hermitian-Einstein connections on  $\mathbb{E}^{4k}$ . We show that the eight-dimensional solutions preserve  $3/16$  of the ten-dimensional  $N=1$  supersymmetry. We argue that under the correspondence between the BPS states of Yang-Mills theory and those of M-theory that arises in the context of Matrix models, the instantons at angles configuration corresponds to the longitudinal intersecting 5-branes on a string at angles configuration of M-theory.

## 1. Introduction

Instantons are Yang-Mills configurations which are characterized by the first Pontryagin number,  $\nu$ , and obey the antiself-duality condition  $\star F = -F$ , where  $F$  is the field strength of a gauge potential  $A$ , the star is the Hodge star and  $\star\star = 1$  for Euclidean signature 4-manifolds. One key property of this condition is that together with the Bianchi identities,  $\nabla_{[M}F_{NL]} = 0$ , it *implies* the field equations of the Yang-Mills theory. At first, a large class of instanton solutions was found using the ansatz of [1, 2]. Later a systematic classification of the solutions of the antiself-duality conditions was done in [3], which has become known as the ADHM construction. Subsequently, many generalizations of the self-duality condition beyond four dimensions have been proposed and for some of them ADHM-like constructions have been found [4, 5, 6].

The understanding of non-perturbative aspects of various superstring theories involves the investigation of configurations which have the interpretation of intersecting branes. In the context of the effective theory, these are extreme solutions of various supergravity theories [7]. Such solutions are constructed using powerful superposition rules from the ‘elementary’ brane solutions that preserve 1/2 of the spacetime supersymmetry (for recent work see [8, 9] and references within). One feature of the intersecting branes is that the intersections can occur at angles in such a way that the configurations preserve a proportion of supersymmetry [10]. Solutions of various supergravity theories that have the interpretation of intersecting branes at angles have been found in [11-16].

More recently, a non-perturbative formulation of M-theory and string theory was proposed using the Matrix models [17, 18, 19, 20]. These involve the study of certain Yang-Mills theories on a torus which is dual to the compactification torus of M-theory. The U-duality groups [21] of IIA superstring theory are naturally realised in the context of matrix models [22]. As a result, there is a cor-

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\* We have chosen the antiself-duality condition in the definition of the instanton but our results can be easily adapted to the self-duality one.

correspondence between the BPS states of these Yang-Mills theories [23] and those of M-theory and superstring theories. In the low energy effective theory, the BPS states of the latter can be described by various classical solitonic solutions of various supergravity theories. This indicates that there is a correspondence between BPS states of Yang-Mills theories with the solitonic solutions of supergravity theories. Since in some cases, the solitonic BPS states of Yang-Mills theories can be described by classical solutions that carry the appropriate charges, the above argument suggests that there is a correspondence between solitonic solutions of supergravity theory with solitonic solutions of Yang-Mills theory. Although this correspondence is established for a compactification torus of finite size, we expect that the correspondence between the solutions of the Yang-Mills theory and supergravity theory will persist in the various limits where the torus becomes very large or very small. One example of such correspondence is the following: The longitudinal five-brane of the Matrix model is described either as the five-brane solution of D=11 supergravity superposed with a pp-wave and compactified on a four-torus or as an instanton solution of the Yang-Mills theory (see for example [22, 23]) on the dual four-torus. (The pp-wave carries the longitudinal momentum of the five-brane.) Ignoring the size of the compactifying torus, one can simply say that there is a correspondence between longitudinal five-branes and instantons. One purpose of this letter is to demonstrate the correspondence between BPS solutions of the Yang-Mills theories with BPS solutions of the supergravity theories with another example. In D=11 supergravity theory, there is a configuration with the interpretation of intersecting five-branes at angles [11] on a string. This configuration is associated with the group  $Sp(2)$ ; the proportion of supersymmetry preserved is directly related to the number of singlets in the decomposition of the spinor representation in eleven dimensions under  $Sp(2)$ . Superposing this configuration with a pp-wave along the string directions corresponds to a Matrix theory configuration with the interpretation of *intersecting longitudinal fivebranes at angles*. Now using the correspondence between the longitudinal fivebranes and the Yang-Mills instantons mentioned above, one concludes that there should exist

solutions of Yang-Mills theory that have the interpretation of *instantons at angles*. Moreover since the instanton configuration must preserve the same proportion of supersymmetry as the supergravity one, it must also be associated with the group  $Sp(2)$ .

To superpose Yang-Mills BPS configurations, like for example Yang-Mills instantons or monopoles, at angles, we embed them as solutions of an appropriate higher-dimensional Yang-Mills theory. Such solutions will be localized on a subspace of the associated spacetime. Then given such a configuration we shall superpose it with another similar one which, however, may be localized in a different subspace. We require that the solutions of the Yang-Mills equations which have the interpretation of BPS configurations at angles to have the following properties: (i) They solve a BPS-like equation which reduces to the standard BPS conditions of the original BPS configurations in the appropriate dimension (ii) if the solution which describes the superposition can be embedded in a supersymmetric theory, then it preserves a proportion of supersymmetry.

In this letter, we shall describe the superposition of four-dimensional instantons. Since instantons are associated with four-planes, it is clear that the configuration that describes the superposition of two instantons at an angle is eight-dimensional; in general the configuration for  $k$  instantons lying on linearly independent four-planes is  $4k$ -dimensional. It turns out that the appropriate BPS condition in  $4k$  dimensions necessary for the superposition of four-dimensional instantons has been found by Ward [5]. Here we shall describe how this BPS condition is associated with the group  $Sp(k)$ . This allows us to interpret a class of the  $4k$ -dimensional instanton solutions of Corrigan, Goddard and Kent [6] as four-dimensional instantons at angles. We shall find that there *four angles* associated with the superposition of each pair of two four-dimensional instantons which depend on the ADHM parameters. The eight-dimensional solutions preserve  $3/16$  of the  $N=1$  ten-dimensional supersymmetry and correspond to the intersecting longitudinal fivebranes at angles configuration of the eleven-dimensional supergravity.

## 2. Tri-Hermitian-Einstein Connections

The BPS condition of Ward [5] is naturally associated with the group  $Sp(k)$ . For this, we first observe that the curvature

$$F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N] , \quad (1)$$

of a connection  $A$  on  $\mathbb{E}^{4k}$  can be thought as an element of  $\Lambda^2(\mathbb{E}^{4k})$  with respect to the space indices. Now  $\Lambda^2(\mathbb{E}^{4k}) = so(4k)$  where  $so(4k)$  is the Lie algebra of  $SO(4k)$ . Decomposing  $so(4k)$  under  $Sp(k) \cdot Sp(1)$ , we find that

$$\Lambda^2(\mathbb{E}^{4k}) = sp(k) \oplus sp(1) \oplus \lambda_0^2 \otimes \sigma^2 \quad (2)$$

where we have set  $\Lambda^1(\mathbb{E}^{4k}) = \lambda \otimes \sigma$  under the decomposition of one forms under  $Sp(k) \cdot Sp(1)$ ,  $\lambda_0^2$  is the symplectic traceless two-fold anti-symmetric irreducible representation of  $Sp(k)$  and  $\sigma^2$  is the symmetric two-fold irreducible representation of  $Sp(1)$ . The integrability condition of [5] can be summarized by saying that the components of the curvature along the last two subspaces in the decomposition (2) vanish. This implies that the space indices of the curvature take values in  $sp(k)$ . It is convenient to write this BPS condition in a different way. Let  $\{I_r; r = 1, 2, 3\}$  be three complex structures in  $\mathbb{E}^{4k}$  that obey the algebra of imaginary unit quaternions, *i.e.*

$$I_r I_s = -\delta_{rs} + \epsilon_{rst} I_t . \quad (3)$$

The BPS condition then becomes

$$(I_r)^P{}_M (I_r)^R{}_N F_{PR}{}^a{}_b = F_{MN}{}^a{}_b , \quad (4)$$

(no summation over  $r$ ), *i.e.* the curvature is (1,1) with respect to all complex structures. We shall refer to the connections that satisfy this condition as tri-Hermitian-Einstein connections. We remark that this is precisely the condition required by (4,0) supersymmetry on the curvature of connection of the Yang-Mills sector in two-dimensional sigma models<sup>★</sup> [24, 25, 26].

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★ For the application of the instantons of [6] in sigma models see [27].

It is straightforward to show that (4) reduces to the antiself-duality condition in four dimensions<sup>†</sup> and that it *implies* the Yang-Mills field equations in  $4k$  dimensions. Moreover any solution of (4) is an example of an Hermitian-Einstein (or Hermitian Yang-Mills) connection<sup>‡</sup> (see for example [28, 29]). The latter are connections for which (i) the curvature tensor is (1,1) with respect to a complex structure and (ii)

$$g^{\alpha\bar{\beta}}F_{\alpha\bar{\beta}} = 0 , \quad (5)$$

where  $g$  is a hermitian metric. We remark that the above two conditions for Hermitian-Einstein connections on  $\mathbb{R}^{2n}$  imply that the non-vanishing components of the curvature are in  $su(n)$ . (The condition (5) together with the Bianchi identity imply the Yang-Mills field equations.) The connections that satisfy (4) are Hermitian-Einstein with respect to three different complex structures.

### 3. One-angle Solutions

To interpret some of the solutions of [5] and [6] as four-dimensional instantons at angles, we first use an appropriate generalization of the t'Hooft ansatz. For this, we write  $\mathbb{E}^{4k} = \mathbb{E}^4 \otimes \mathbb{E}^k$ , i.e. we choose the coordinates

$$\{y^M\} = \{x^{i\mu}; i = 1, \dots, k; \mu = 0, 1, 2, 3\} , \quad (6)$$

on  $\mathbb{E}^{4k}$ . Then we introduce three complex structures on  $\mathbb{E}^{4k}$

$$(\mathbf{I}_r)^{i\mu}{}_{j\nu} = I_r{}^\mu{}_\nu \delta^i_j \quad (7)$$

where  $\{I_r; r = 1, 2, 3\}$  are three complex structures on  $\mathbb{E}^4$  associated with the

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<sup>†</sup> In fact it reduces to either the self-duality or the antiself-duality condition depending on the choice of orientation of the four-manifold.

<sup>‡</sup> For  $k > 1$ , the Hermitian-Einstein BPS condition is weaker than that of (4).

Kähler forms

$$\begin{aligned}\omega_1 &= dx^0 \wedge dx^1 + dx^2 \wedge dx^3 \\ \omega_2 &= -dx^0 \wedge dx^2 + dx^1 \wedge dx^3 \\ \omega_3 &= dx^0 \wedge dx^3 + dx^1 \wedge dx^2 ,\end{aligned}\tag{8}$$

respectively. Choosing the volume form as  $\epsilon = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ , these form a basis of self-dual 2-forms in  $\mathbb{E}^4$ . We also write the Euclidean metric on  $\mathbb{E}^{4k}$  as

$$ds^2 = \delta_{\mu\nu} \delta_{ij} dx^{i\mu} dx^{j\nu} .\tag{9}$$

Similarly, the curvature two-form is

$$F_{MN} = F_{i\mu j\nu} .\tag{10}$$

Writing

$$F_{i\mu j\nu} = F_{[\mu\nu](ij)} + F_{(\mu\nu)[ij]} ,\tag{11}$$

we find that (4) implies that

$$\begin{aligned}\frac{1}{2} F_{[\mu\nu](ij)} \epsilon^{\mu\nu}{}_{\rho\sigma} &= -F_{[\rho\sigma](ij)} \\ F_{(\mu\nu)[ij]} &= \delta_{\mu\nu} f_{ij} ,\end{aligned}\tag{12}$$

where  $f_{ij}$  is a  $k \times k$  antisymmetric matrix. Next, we shall further assume that the gauge group is  $SU(2) = Sp(1)$  and write the ansatz

$$A_{i\mu} = i \Sigma_\mu{}^\nu \partial_{i\nu} \log \phi(x)\tag{13}$$

where the matrix two-form  $\Sigma_{\mu\nu}$  is the 't Hooft tensor, *i.e.*

$$\Sigma_{\mu\nu} = \frac{1}{2} \omega_{r\mu\nu} \sigma^r ,\tag{14}$$

and  $\{\sigma_r; r = 1, 2, 3\}$  are the Pauli matrices. After some computation, we find that

the field strength  $F$  of  $A$  in (13) satisfies the first equation in (12) provided that

$$\frac{1}{\phi} \partial_i \cdot \partial_j \phi = 0 . \quad (15)$$

This is a harmonic-like equation and a solution is

$$\phi = 1 + \frac{\rho^2}{(p_i x^{i\mu} - a^\mu)^2} , \quad (16)$$

where  $\{p\} = (p_1, p_2, \dots, p_k)$  are  $k$  real numbers,  $a^\mu$  is the centre of the harmonic function and  $\rho^2$  is the parameter associated with the size of the instanton. More general solutions can be obtained by a linear superposition of the above solution for different choices of  $\{p\}$  and  $a^\mu$  leading to

$$\phi = 1 + \sum_{\{p\}} \sum_a \frac{\rho_a^2(\{p\})}{(p_i x^{i\mu} - a^\mu(\{p\}))^2} . \quad (17)$$

Finally, it is straightforward to verify that the curvature of the connection (13) with  $\phi$  given as in (17), satisfies (4).

To investigate the properties of the above solutions, let us suppose that the harmonic function  $\phi$  involves only one  $\{p\}$  as in (16). In this case it is always possible, after a coordinate transformation, to set  $\{p\} = \{1, 0, \dots, 0\}$ . Then  $\phi$  becomes a harmonic function on the ‘first’ copy of  $\mathbb{E}^4$  in  $\mathbb{E}^{4k}$ . The resulting configuration is that derived from the ansatz of [1, 2] on  $\mathbb{E}^4$ . In particular, the instanton number is equal to the number of centres  $\{a^\mu\}$  of the harmonic function. In this case the singularities at the centres of the harmonic function can be removed with a suitable choice of a gauge transformation [30]. Next, let us suppose that our solution involves two linearly independent vectors  $\{p\}$ , say  $\{p\} = \{p\}$  and  $\{p\} = \{q\}$ . If we take  $|p_i x^i| \rightarrow \infty$ , then the configuration reduces to the solution associated with a harmonic function that involves only  $\{q\}$ . As we have explained above, this is the instanton solution of [1, 2] on an appropriate 4-plane in  $\mathbb{E}^{4k}$ . Similarly if we take  $|q_i x^i| \rightarrow \infty$ , then the configuration reduces to the solution associated with the



harmonic function that involves only  $\{p\}$ . We can therefore interpret this solution as the superposition of two four-dimensional instantons at angles in  $\mathbb{E}^{4k}$ . The angle is

$$\cos \theta = \frac{p \cdot q}{|p||q|} . \quad (18)$$

As it may have been expected from the non-linearity of the gauge potentials in (4), this superposition is non-linear even in the case for which  $\{p\}$  and  $\{q\}$  are orthogonal. The solution appears to be singular at the positions of the harmonic function. For simplicity let us consider two instantons at angles with instanton number one. In this case we have two centres. The singular set is defined by the two codimension-four-planes

$$\begin{aligned} p_i x^{i\mu} - a^\mu &= 0 \\ q_i x^{i\mu} - b^\mu &= 0 , \end{aligned} \quad (19)$$

in  $\mathbb{E}^8$ . We have investigated the singularity structure of the solution when  $\{p\}$  and  $\{q\}$  are orthogonal using a method similar to [30]. We have found that these singularities can be removed everywhere apart from the intersection of the two singular sets. A similar observation has been made in [6]. To conclude, let us briefly consider the general case. The number of four dimensional instantons involved in the configuration is equal to the number of linearly independent choices for  $\{p\}$ . Their instanton number is equal to the number of centres associated with each choice of  $\{p\}$ . The general solution describes the superposition of these four-dimensional instantons at angles.

## 4. Four-angle solutions

The ADHM ansatz of [6] describes solutions with more parameters than those found in the previous section using the t'Hooft ansatz. Here we shall interpret some of these parameters as angles. In fact we shall find that there are *four angles* between each pair of planes associated with these solutions all determined in terms of ADHM parameters. For simplicity, we shall take  $Sp(1)$  as the gauge group. The ADHM ansatz is

$$A = v^\dagger dv, \quad (20)$$

where  $v$  is an  $(\ell + 1) \times 1$  matrix of quaternions normalized as  $v^\dagger v = 1$ , and  $\ell$  is an integer which is identified with the ‘total’ instanton number (see [6]). The matrix  $v$  satisfies the condition

$$v^\dagger \Delta \equiv v^\dagger (a + b_i x^i) = 0 \quad (21)$$

where  $a, b_i$  are  $(\ell + 1) \times \ell$  matrices of quaternions; we have arranged the  $4k$  coordinates into quaternions  $\{x^i; i = 1, \dots, k\}$  ( $\mathbb{E}^{4k} = \mathbb{H}^k$ ). The connection (20) satisfies the condition (4) provided that the matrices  $a^\dagger a, b_i^\dagger b_j, a^\dagger b_i$  are symmetric as matrices of quaternions.

A convenient choice of matrices  $a, b_i$  leads to

$$\begin{aligned} (\Delta)_{0n} &= -a_0 \lambda_n \\ (\Delta)_{nm} &= (p_{ni}^\dagger x^i - a_n) \delta_{nm} \end{aligned} \quad (22)$$

where  $\{\lambda_n; n = 1, \dots, \ell\}$  are real numbers, and  $\{p_{ni}; n = 1, \dots, \ell; i = 1, \dots, k\}$  and  $\{a_0, a_n; n = 1, \dots, \ell\}$  are quaternions. The solutions discussed in the previous section using the t'Hooft ansatz correspond to choosing  $\{p_{ni}; n = 1, \dots, \ell, i = 1, \dots, k\}$  to be real numbers. For  $v$ , we find

$$\begin{aligned} (v)_0 &= -\frac{a_0}{|a_0|} f^{-1} \\ (v)_n &= -|a_0| \lambda_n [(x^i)^\dagger p_{ni} - a_n^\dagger]^{-1} f^{-1} \end{aligned} \quad (23)$$

where  $f$  is the normalization factor

$$f^2 = 1 + |a_0|^2 \sum_{n=1}^{\ell} \frac{\lambda_n^2}{|p_{ni}^\dagger x^i - a_n|^2} . \quad (24)$$

We remark that the sum in the normalization factor is over the centres  $\{a_n; n = 1, \dots, \ell\}$ . It is clear from the form of the normalization factor that this solution is associated with  $\ell$  codimension-four-planes in  $\mathbb{E}^{4k}$ . The equations of these planes are

$$p_{ni}^\dagger x^i - a_n = 0 , \quad (25)$$

for  $n = 1, \dots, \ell$ . The four normalized normal vectors to these planes are

$$\mathcal{N}_n = \frac{1}{|p_n|} (p_n^i)^\dagger \frac{\partial}{\partial x^i} , \quad (26)$$

in quaternionic notation, where  $|p_n|^2 = \delta_{ij} (p_n^i)^\dagger p_n^j$ . Next let us consider two such codimension-four-plane determined say by  $p = p_1$  and  $q = p_2$  and with associated normal vectors  $\mathcal{N}$  and  $\tilde{\mathcal{N}}$ , respectively. The angles associated with these planes are given by the inner product of their normal vectors, so

$$\cos(\theta) = (\tilde{\mathcal{N}})^\dagger \mathcal{N} = \frac{\delta_{ij} (q^i)^\dagger p^i}{|p||q|} , \quad (27)$$

where  $\cos(\theta)$  is a quaternion in the obvious notation. There are four angles because  $\cos(\theta)$  has four components. These four angles are all different for a generic choice of  $p, q$  and independent from the choice of gauge fixing for the residual symmetries of the ADHM construction [6]. It is worth mentioning that each pair of four-dimensional instantons are superposed at  $Sp(2)$  angles. For this we observe that the normal vectors (26) of the two codimension-four-planes span an eight-dimensional subspace in  $\mathbb{E}^{4k}$  and their coefficients are quaternions. So, they can be related with an  $Sp(2)$  rotation (after choosing a suitable basis in  $\mathbb{E}^{4k}$ ). As a result the two four-planes spanned by the normal vectors in  $\mathbb{E}^8$  are at  $Sp(2)$  angles. Such

four-planes are parameterized by the four-dimensional coset space  $Sp(2)/SO(4)$ ; this explains the presence of four angles in (27). It also suggests that there may be a generalization of the solutions of [11] to describe the superposition five-branes and KK-monopoles depending on four angles.

A naive counting of the dimension of the moduli of the 4k-dimensional solutions which takes account of the dimension of moduli of 4-dimensional instantons involved in the superposition and their associated angles *does not* reproduce the dimension of the moduli space in [6]. However, all their solutions have a decaying behaviour at large distances consistent with the interpretation that they are four-dimensional instantons at angles.

## 5. Supersymmetry

Among the above Yang-Mills configurations on  $\mathbb{E}^{4k}$  only the instantons at angles in  $\mathbb{E}^8$  and the instantons on  $\mathbb{E}^4$  are solutions of ten-dimensional supersymmetric Yang-Mills theory preserving a proportion of supersymmetry. The supersymmetry condition is

$$F_{MN}\Gamma^{MN}\epsilon = 0 \ , \tag{28}$$

where  $\epsilon$  is the supersymmetry parameter. In the four-dimensional case the instantons preserve 1/2 of the supersymmetry. In the eight-dimensional case condition (4) implies that  $F_{MN}$  is in  $sp(2)$ , an argument similar to that in [11] can be used to show that the instantons at angles in  $\mathbb{E}^8$  will preserve 3/16 of supersymmetry. As a solution of the Matrix theory, the eight-dimensional instantons at angles preserve 3/32 of spacetime supersymmetry. This is in agreement with the proportion of supersymmetry preserved by the intersecting five-branes on a string at angles and superposed with a pp-wave solution of D=11 supergravity. However there is a difference. In the case of two orthogonal intersecting five-branes on a string with a pp-wave superposed the proportion of the supersymmetry preserved is 1/8 but this is not the case for the configuration of two orthogonal instantons. To see this,

first observe that the solutions of the previous two sections will preserve 1/4 of the N=1 ten-dimensional supersymmetry if the components of the curvature are in the  $sp(1) \oplus sp(1)$  subalgebra of  $sp(2)$ . However this is not so because a direct calculation reveals that the mixed components  $F_{\mu 1, \nu 2}$  of the curvature tensor do not vanish. Further we remark that one might have thought that it is possible to superpose two four-dimensional instantons  $B_\mu$  and  $C_\alpha$  with gauge group  $G$  localised at two orthogonal four-planes in  $\mathbb{E}^8$  by simply setting  $A = (B, C)$  and requiring that the gauge group of the new connection is again  $G$ . However, this is not a solution of the BPS condition (4) for a non-abelian gauge group in eight-dimensions. This is because (4) is non-linear in the connection. However the above linear superposition is a solution if the gauge groups of  $B$  and  $C$  are treated independently, i.e. if the gauge group of  $A$  is  $G \times G$ .

## 6. Concluding Remarks

We have interpreted some of the 4k-dimensional instantons of [5] and [6] as superposition of four-dimensional instantons at angles. We have shown that there *four angles* associated with the superposition of two four-dimensional instantons which depend on the ADHM parameters. These solutions are examples of Hermitian-Einstein connections in 4k dimensions and the eight-dimensional ones preserve 3/16 of the N=1 ten-dimensional supersymmetry. Because of the close relation between the self-duality condition and the BPS equations for magnetic monopoles we expect that one can generalize the above construction to find solutions of Yang-Mills equations on  $\mathbb{E}^{3k}$  which have the interpretation of monopoles at angles. In addition, the Yang-Mills configurations of [6] and the HKT geometries described in [31] combined give new examples of two-dimensional supersymmetric sigma models with (4,0) supersymmetry. These provide consistent backgrounds for the propagation of the heterotic string [32]. Another application of the instantons of [6] is in the context of D-branes. In IIA theory, they are associated with the D-brane bound state of a 0-brane within a 4-brane within an 8-brane, and in the

IIB theory they are associated with the D-brane bound state of a D-string within a D-5-brane within a D-9-brane.

There are many other ways to superpose two or more four-dimensional instantons. Here we have described the superposition using the condition that the components of the curvature  $F$  of the Yang-Mills connection are in  $sp(k)$ . Another option is to use the Hermitian-Einstein condition associated with  $su(2k)$  which we have already mentioned in section two. In eight dimensions this will lead to configurations preserving  $1/8$  of the  $N=1$  ten-dimensional supersymmetry. In addition in eight dimensions one can allow the components of  $F$  to lie in  $spin(7)$ . This will result in superpositions of instantons preserving  $1/16$  of the  $N=1$  ten-dimensional supersymmetry. In fact, some  $spin(7)$  instanton solutions have already been found [33]. It will be of interest to see whether they can be interpreted as four-dimensional instantons at angles. Since  $Sp(2)$  is a subgroup of  $Spin(7)$ , the solutions of [6] are also examples of  $spin(7)$  instantons, albeit of a particular type. There are more possibilities in lower dimensions, for example the  $G_2$  instantons in seven dimensions [34, 35] which may have a similar interpretation.

#### NOTE ADDED

While we were revising our work, the paper [36] by N. Ohta and J-G Zhou appeared in which the relation between supergravity configurations and Yang-Mills ones is also discussed. However, they use constant Yang-Mills configurations on the eight torus.

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